Unit 103: Further Mathematics for Engineering Technicians

Unit code: H/600/0280

QCF Level 3: BTEC Nationals

Credit value: 10

Guided learning hours: 60

Aim and purpose

This unit aims to enhance learners' knowledge of the mathematical principles used in engineering, enabling them to pursue further study on a higher education qualification in engineering.

Unit introduction

Mathematics is an essential tool for any electrical or mechanical engineering technician. With this in mind, this unit emphasises the engineering application of mathematics. For example, learners could use an integral calculus method to obtain the root mean square (RMS) value of a sine wave over a half cycle.

The first learning outcome will extend learners' knowledge of graph plotting and will develop the technique of using a graph to solve (find the roots of), for example, a quadratic equation.

Learning outcome 2 involves the use of both arithmetic and geometric progressions for the solution of practical problems. The concept of complex numbers, an essential tool for electrical engineers considering, is also introduced.

Learning outcome 3 considers the parameters of trigonometrical graphs and the resultant wave when two are combined. The use of mathematical formulae in the latter half of this learning outcome enables a mathematical approach to wave combination to be considered.

Finally, in learning outcome 4, calculus techniques are further developed and used to show their application in engineering.

Learning outcomes

On completion of this unit a learner should:

1 Be able to use advanced graphical techniques

2 Be able to apply algebraic techniques

3 Be able to manipulate trigonometric expressions and apply trigonometric techniques

4 Be able to apply calculus.

Unit content

1 Be able to use advanced graphical techniques

Advanced graphical techniques: graphical solution eg of a pair of simultaneous equations with two unknowns, to find the real roots of a quadratic equation, for the intersection of a linear and a quadratic equation, non-linear laws such as ($y = ax^2 + b,\ \ y = a + \frac{b}{x}$), by the use of logarithms to reduce laws of type $y = ax^n$ to straight line form, of a cubic equation such as $2x^3 - 7x^2 + 3x + 8 = 0$, recording, evaluating and plotting eg manual, computerised

2 Be able to apply algebraic techniques

Arithmetic progression (AP): first term (a), common difference (d), nth term eg $a + (n - 1)d$; arithmetic series eg sum to n terms, $S_n = \frac{n}{2}\{2a + (n-1)d\}$

Geometric progression (GP): first term (a), common ratio (r), nth term eg $a\,r^{\,n-1}$; geometric series eg sum to n terms, $S_n = \dfrac{a(r^n - 1)}{r - 1}$, sum to infinity $S_\infty = \dfrac{a}{1-r}$; solution of practical problems eg compound interest, range of speeds on a drilling machine

Complex numbers: addition, subtraction, multiplication of a complex number in Cartesian form, vector representation of complex numbers, modulus and argument, polar representation of complex numbers, multiplication and division of complex numbers in polar form, polar to Cartesian form and vice versa, use of calculator

Statistical techniques: review of measure of central tendency, mean, standard deviation for ungrouped and grouped data (equal intervals only), variance

3 Be able to manipulate trigonometric expressions and apply trigonometric techniques

Trigonometrical graphs: amplitude, period and frequency, graph sketching eg $\sin x$, $2 \sin x$, $\frac{1}{2} \sin x$, $\sin 2x$, $\sin \frac{1}{2} x$ for values of x between 0 and 360°; phase angle, phase difference; combination of two waves of the same frequency

Trigonometrical formulae and equations: the compound angle formulae for the addition of sine and cosine functions eg $\sin (A \pm B)$; expansion of $R \sin (\mathrm{wt} + \alpha)$ in the form a $\cos \mathrm{wt}$ + b $\sin \mathrm{wt}$ and vice versa

4 Be able to apply calculus

Differentiation: review of standard derivatives, differentiation of a sum, function of a function, product and quotient rules, numerical values of differential coefficients, second derivatives, turning points (maximum and minimum) eg volume of a rectangular box

Integration: review of standard integrals, indefinite integrals, definite integrals eg area under a curve, mean and RMS values; numerical eg trapezoidal, mid-ordinate and Simpson's rule

Assessment and grading criteria

In order to pass this unit, the evidence that the learner presents for assessment needs to demonstrate that they can meet all the learning outcomes for the unit. The assessment criteria for a pass grade describe the level of achievement required to pass this unit.

Assessment and grading criteria		
To achieve a pass grade the evidence must show that the learner is able to:	**To achieve a merit grade the evidence must show that, in addition to the pass criteria, the learner is able to:**	**To achieve a distinction grade the evidence must show that, in addition to the pass and merit criteria, the learner is able to:**
P1 use a graphical technique to solve a pair of simultaneous linear equations [CT5]	**M1** use the laws of logarithms to reduce an engineering law of the type $y = ax^n$ to straight line form, then using logarithmic graph paper, plot the graph and obtain the values for the constants a and n	**D** using a graphical technique determine the single wave resulting from a combination of two waves of the same frequency and then verify the result using trigonometrical formulae
P2 solve a practical engineering problem involving an arithmetical progression	**M2** use complex numbers to solve a parallel arrangement of impedances giving the answer in both Cartesian and polar form	**D2** use numerical integration and integral calculus to analyse the results of a complex engineering problem.
P3 solve a practical engineering problem involving geometric progression	**M3** use differential calculus to find the maximum/minimum for an engineering problem.	
P4 perform the two basic operations of multiplication and division to a complex number in both rectangular and polar form, to demonstrate the different techniques		
P5 calculate the mean, standard deviation and variance for a set of ungrouped data [IE4]		
P6 calculate the mean, standard deviation and variance for a set of grouped data [IE4]		
P7 sketch the graph of a sinusoidal trigonometrical function and use it to explain the terms and describe amplitude, periodic time and frequency		

Assessment and grading criteria		
To achieve a pass grade the evidence must show that the learner is able to:	**To achieve a merit grade the evidence must show that, in addition to the pass criteria, the learner is able to:**	**To achieve a distinction grade the evidence must show that, in addition to the pass and merit criteria, the learner is able to:**
P8 use two of the compound angle formulae and verify their relationship		
P9 find the differential coefficient for three different functions to demonstrate the use of function of a function and the product and quotient rules		
P10 use integral calculus to solve two simple engineering problems involving the definite and indefinite integral.		

PLTS: This summary references where applicable, in the square brackets, the elements of the personal, learning and thinking skills applicable in the pass criteria. It identifies opportunities for learners to demonstrate effective application of the referenced elements of the skills.

Key	IE – independent enquirers CT – creative thinkers	RL – reflective learners TW – team workers	SM – self-managers EP – effective participators

Essential guidance for tutors

Delivery

This unit should be delivered at a later stage in the course, after a suitable foundation in mathematics and engineering principles has been established.

Every opportunity should be taken to apply and contextualise the underpinning mathematical principles. Tutors could provide a selection of well-prepared, vocationally relevant examples and assignments that are tailored to area-specific programmes of study, as well as selecting specific applications from the suggested option.

Regular opportunities (for example classroom exercises) to address the relevant techniques should be provided as part of formative assessment. Constant feedback, using additional formative tests and coursework that falls outside the formal summative assessment, may be used to aid learning without necessarily being graded.

The unit content does not need to be taught or assessed in order and it is left to centres to decide on their preferred order of delivery.

Note that the use of 'eg' in the content is to give an indication and illustration of the breadth and depth of the area or topic. As such, not all content that follows an 'eg' needs to be taught or assessed.

Outline learning plan

The outline learning plan has been included in this unit as guidance and can be used in conjunction with the programme of suggested assignments.

The outline learning plan demonstrates one way in planning the delivery and assessment of this unit.

Topic and suggested assignments/activities and/assessment
Whole-class teaching: • introduction to unit content, scheme of work and assessment requirements • plotting of straight line graphs, choice of axes and explain how to rearrange equations into straight line form • tutor-led demonstration of plotting of straight line graphs with a minimum of three points to obtain point of intersection • discuss the shape of the graphical plot of a quadratic equation and the effects of the various parameters • explain the formation of a table of values with possible use of computer to assist and then demonstrate how to plot smooth curves and the application to find the roots of a quadratic equation. *Individual learner activity*: • tutor-led exercises on graphical solution of simultaneous equations • tutor-led exercises on the graphical solution of quadratic equation. *Whole-class teaching*: • explain how by plotting a quadratic and linear graph simultaneously the intersection can be projected onto the x-axis to give the roots of an equation, eg Plot the graph $y = 2x^2$ of and use the graph to solve the equation $2x^2 + 3x - 4 = 0$ • explain how laws which are not apparently linear can be reduced to give a straight line relationship and then demonstrate the reduction of typical examples, eg $y = ax^2 + b$, plot y against x^2 • revise the laws of logarithms and explain how logarithms can be used to reduce laws of the type $y = ax^n$ to a straight line form. *Individual learner activity*: • tutor-led involvement in the graphical solution of problems involving non-linear laws • tutor-led exercises of graphical solutions tutor-led demonstration of the application to a variety of problems • tutor-led solution of graphical problems using ordinary graph paper. *Whole-class teaching*: • to review the previous week's work and demonstration of solution of laws of the type $y = ax^n$ but using logarithmic graph paper • explain and demonstrate the differing type of cubic equation, eg 1 root, 2 roots, 3 roots • show how a table can be built up manually and computer generated and demonstrate plotting of a cubic equation • summarise all the previous weeks' work using manual and graphical techniques. *Individual learner activity*: • tutor-led exercises on graphical solutions using logarithmic graph paper • tutor-led solution of cubic equations graphically • tutor-led exercises in graphical solutions of all techniques taught in previous weeks using both manual and computerised methods.
Preparation for and carrying out **Assignment 1: Graphical Techniques** (P1, M1, D1).

Topic and suggested assignments/activities and/assessment
Whole-class teaching: • define an arithmetic progression and express an arithmetic progression in terms of the first term (a) and the common difference (d) • explain the expression for the nth term and the sum to n terms • define a geometric progress and express a geometric progression in terms of the first term (a) and the common ratio (r) • explain the expression for the nth term, the sum to n terms and the sum to infinity. *Individual learner activity*: • tutor-led solution of problems involving arithmetic progressions • tutor-led solution of practical problems on geometric progressions. *Whole-class teaching*: • introduce the idea of a complex number and explain the Argand diagram • demonstration of the algebra of complex numbers in Cartesian form • introduce the polar form of a complex number and demonstrate the algebra of complex numbers in polar form. *Individual learner activity*: • tutor-led solution of problems on complex numbers in Cartesian form tutor-led • solution of problems on complex numbers in polar form. *Whole-class teaching*: • discuss the relationship between the two forms of complex number and demonstrate the use of a calculator to interchange between the two forms of a complex number • explain the application of complex numbers in practical situations, eg alternating current theory and mechanical vector analysis • review the evaluation of the mean, median and mode for discrete and grouped data • explain the significance of standard deviation and variance and demonstrate the evaluation of standard deviation and variance for ungrouped data. *Individual learner activity*: • tutor-led solution of practical problems involving complex numbers • tutor-led solution of problems involving standard deviation and variance. *Whole-class teaching*: • demonstrate the evaluation of standard deviation and variance for grouped data and show how to use the statistical functions on calculators to obtain the mean, standard deviation and variance. *Individual learner activity*: • tutor-led solution of problems involving standard deviation and variance.
Preparation for and carrying out **Assignment 2: Algebraic Techniques** (P2, P3, P4, P5, P6, M2).

Topic and suggested assignments/activities and/assessment
Whole-class teaching: • to revise the relationship between degrees and radians • explain amplitude, periodic time and frequency and demonstrate the concept of graph sketching for periodic functions • discuss shapes of $\sin x$, $2 \sin x$, $½ \sin x$, $\sin 2x$ for values of x between 0 and 360° by using Powerpoint demonstration. *Individual learner activity*: • tutor-led graph sketching exercises on graph paper and computer. *Whole-class teaching*: • using Powerpoint to demonstrate the concepts of phase angle and phase difference followed by a demonstration of how a single wave results from a combination of two waves of the same frequency using graphs and phasor addition • introduce the compound angle formulae and demonstrate its use followed by an explanation of the expansion of $R \sin (\mathrm{wt} + \alpha)$ in the form a $\cos \mathrm{wt} + \mathrm{b} \sin \mathrm{wt}$ and then lead learners in solution of simple problems *Individual learner activity*: • tutor-led exercises on graphical plots of the combination of two waves • tutor-led solution of problems on trigonometrical formula and equations.
Preparation for and carrying out **Assignment 3: Trigonometric Techniques** (P7, P8).
Whole-class teaching: • revise the concept of differentiation and review of standard derivatives • demonstration of the differentiation of different functions, including polynomial, exponential and trigonometrical • differentiation of various combinations of sums • demonstrate methods of differentiation of functions using function of a function, product and quotient rules. *Individual learner activity*: • tutor-led exercises on differentiation • tutor-led exercises in differentiation using the various rules. *Whole-class teaching*: • demonstrate the evaluation of differential coefficients and introduce the concept of successive differentiation and application, eg velocity and acceleration. *Individual learner activity*: • tutor-led exercises on different applications of differentiation. *Whole-class teaching*: • introduce the idea of stationary points and show how to evaluate maximum and minimum values for a range of functions • demonstrate calculation of maximum/minimum values for practical applications. *Individual learner activity*: • tutor-led evaluation of maximum and minimum values.

Topic and suggested assignments/activities and/assessment
Whole-class teaching: • revise the concept of integration and review of standard integrals, indefinite integration, constant of integration and definite integration • demonstrate evaluation of definite integrals and application eg area under a curve. *Individual learner activity*: • tutor-led exercises on integration. *Whole-class teaching*: • demonstrate the application of integration to calculate the mean (average) values • demonstrate the application of integration to calculate the root mean square (rms) values. *Individual learner activity*: • tutor-led evaluation of mean and rms values using integration. *Whole-class teaching*: • explain the three methods for finding areas of irregular shapes • discuss the idea of linking the three methods and comparing with integration to find the area under a curve. *Individual learner activity*: • tutor-led exercises on numerical integration with computer software to assist tabulation and evaluation.
Preparation for and carrying out **Assignment 4: Calculus Applications** (P9, P10, M3, D2).
Feedback on all assessment tasks with guidance on remedial action if necessary. Unit evaluation and close.

Assessment

P1 and M1 are probably best assessed through an assignment with learners being given different equations for a meaningful task (for example two operatives producing a certain number of assemblies) for P1.

For M1, learners will need to provide evidence that they can reduce an engineering law (for example gas pressure and temperature $T = \alpha p^y$, current and voltage $I = VR^k$) to straight line form, then use logarithmic graph paper to plot the graph and obtain values for the constants. This is probably best achieved with an assignment in which learners are each given different values.

P2 and P3 can be combined into one assignment, again relevant to an engineering problem (for example the drilling of bore holes for an arithmetic progression solution and the calculation of drill speeds for a geometric progression solution).

For P4 ,learners could be given different values to demonstrate the two basic operations and this could be linked to M2 to form one assignment.

P5 and P6 could also be linked and assessed through an assignment or short formal test with a relevant application (for example values of resistors, quality control of a product, overtime working).

P7 and P8 could be assessed by a short formal class test. Alternatively, an assignment could be used with different values for the graphical output given to different learners. Either approach would help ensure answers are authentic.

P9, P10 and M3 are possibly best assessed as a short exercise or assignment, with learners being given a list of the standard differential coefficients and integrals to use. For P9, each of the questions could be written to assess all the three rules in turn. P10 requires a simple engineering problem (eg indefinite integral given information to find value of constant and hence required equation, definite integral such as area under a curve). M3 is possibly best linked to P9 and P10 with M3 assessing an engineering problem (eg use of differentiation to find the dimensions of a rectangular box to give the maximum volume).

The merit criteria need to build upon the pass criteria, and as such may form an extension to an assessment containing several of the pass criteria as already indicated.

Evidence for the distinction criteria needs to show that learners have a more in-depth knowledge and understanding. Both the distinction criteria could be assessed by a written assignment. For D1, learners firstly need to use a graphical technique to obtain the single wave resulting from a combination of two waves of the same frequency. Each learner could be given a slightly different equation by varying the values of A and B (eg $V_1 = A\sin\left(100\pi t + \frac{2\pi}{5}\right)$, $V_2 = B\sin\left(100\pi t + \frac{2\pi}{9}\right)$). Learners should be encouraged to use a computer package for recording, evaluating and possibly plotting a range of values (eg t from 0 to 0·02s). By using the double angle formulae (eg $\sin(A \pm B)$) and the expansion of $R\sin(wt \pm \alpha)$ learners could verify their results.

Ideally D2 should provide a comparison between one, two or all three of the numerical integration methods and integration by calculus (eg evaluation of the distance travelled in the first second when a guitar string is plucked about its centre, given an equation for its velocity).

Programme of suggested assignments

The table below shows a programme of suggested assignments that cover the pass, merit and distinction criteria in the assessment and grading grid. This is for guidance and it is recommended that centres either write their own assignments or adapt any Edexcel assignments to meet local needs and resources.

Criteria covered	Assignment title	Scenario	Assessment method
P1, M1, D1	Graphical Techniques	A written activity requiring learners to plot various graphs covering the criteria for outcome one.	A report containing graphical solutions to each of the criteria for outcome one, carried out under controlled conditions or by the use of individual learner values.
P2, P3, P4, P5, P6, M2	Algebraic Techniques	A written activity in which learners are required to perform a number of different algebraic techniques to satisfy the criteria out outcome two.	A report containing the results of calculations to assess the different algebraic techniques covering the criteria for outcome two, carried out under controlled conditions.
P7, P8	Trigonometric Techniques	A written activity requiring learners to carry out calculations and produce graphical evidence to satisfy the two criterion.	A report containing the results of learners' calculations and graphs carried out under controlled conditions.
P9, P10, M3, D2	Calculus Applications	A written activity possibly in examination format to enable learners to demonstrate their ability to use the calculus.	A report containing the solutions to calculations and analysis of calculus applications carried out under controlled conditions.

Links to National Occupational Standards, other BTEC units, other BTEC qualifications and other relevant units and qualifications

This unit forms part of the BTEC Engineering sector suite. This unit has particular links with the following unit titles in the Engineering suite:

Level 1	Level 2	Level 3
	Mathematics for Engineering Technicians	Mathematics for Engineering Technicians
		Electrical and Electronic Principles
		Mechanical Principles and Applications

Unit 4: Mathematics for Engineering Technicians is an essential prerequisite for this unit and as such must be studied prior to this unit.

Essential resources

Learners will need to use an electronic scientific calculator and have access to software packages that support the concepts and principles and their application to engineering.

Employer engagement and vocational contexts

This unit should be delivered in the context of real engineering applications and centres are encouraged to make good use of the contacts that they have with local industry through other, more practical-based engineering units.

There are a range of organisations that may be able help centres engage and involve local employers in the delivery of this unit, for example:

- Work Experience/Workplace learning frameworks – Centre for Education and Industry (CEI, University of Warwick) – www.warwick.ac.uk/wie/cei
- Learning and Skills Network – www.vocationallearning.org.uk
- Network for Science, Technology, Engineering and Maths Network Ambassadors Scheme – www.stemnet.org.uk
- National Education and Business Partnership Network – www.nebpn.org
- Local, regional Business links – www.businesslink.gov.uk
- Work-based learning guidance – www.aimhighersw.ac.uk/wbl.htm

Indicative reading for learners

Textbooks

Bird J – *Engineering Mathematic* (Elsevier Science & Technology, 2007) ISBN 9780750685559

Greer A, Taylor G W – *BTEC National Further Mathematics for Technicians* (Nelson Thornes, 2005) ISBN 9780748794102

Tooley M and Dingle L – *BTEC National Engineering, 2nd Edition* (Newnes, 2007) ISBN 9780750685214

Delivery of personal, learning and thinking skills

The table below identifies the opportunities for personal, learning and thinking skills (PLTS) that have been included within the pass assessment criteria of this unit.

Skill	When learners are …
Independent enquirers	analysing and evaluating statistical information, judging its relevance and value
Creative thinkers	trying out a graphical technique to solve a pair of simultaneous equations and following the idea through.

Although PLTS are identified within this unit as an inherent part of the assessment criteria, there are further opportunities to develop a range of PLTS through various approaches to teaching and learning.

Skill	When learners are …
Reflective learners	reviewing progress when solving problems during the learner's activities and acting on the outcomes to make corrections to understanding/solutions
Team workers	collaborating with others when working on investigative group work to achieve a valid solution
Self-managers	organising time and resources, prioritising actions.

Functional Skills – Level 2

Skill	When learners are …
Mathematics	
Understand routine and non-routine problems in a wide range of familiar and unfamiliar contexts and situations	solving routine electrical and mechanical problems set within engineering contexts and situations
Identify the situation or problem and the mathematical methods needed to tackle it	recognising the relevant parameters and formulae to be applied to given electrical and mechanical situations
Select and apply a range of skills to find solutions	selecting and applying formulae to solve electrical/mechanical problems in engineering
Use appropriate checking procedures and evaluate their effectiveness at each stage	checking the results of solutions to electrical and mechanical problems to evaluate their effectiveness and reality at each stage of the calculation
English	
Speaking and listening – make a range of contributions to discussions and make effective presentations in a wide range of contexts	speaking with and listening to peers and supervisors to establish an understanding of mathematical concepts and issues in engineering
Reading – compare, select, read and understand texts and use them to gather information, ideas, arguments and opinions	selecting, reading and using appropriate mathematical data sources to solve engineering problems
Writing – write documents, including extended writing pieces, communicating information, ideas and opinions, effectively and persuasively	taking notes and solving engineering mathematical problems to communicate accurate solutions effectively.